Statistics: Section B Paper 3

Section B

Q6.

In a region of England, the government decides to use an advertising campaign to encourage people to eat more healthily.

Before the campaign, the mean consumption of chocolate per person per week was known to be 66.5 g, with a standard deviation of 21.2 g

(a) After the campaign, the first 750 available people from this region were surveyed to find out their average consumption of chocolate.

(i) State the sampling method used to collect the survey.

(1)

(ii) Explain why this sample should not be used to conduct a hypothesis test.

(1)

(b) A second sample of 750 people revealed that the mean consumption of chocolate per person per week was 65.4 g

Investigate, at the 10% level of significance, whether the advertising campaign has decreased the mean consumption of chocolate per person per week.

Assume that an appropriate sampling method was used and that the consumption of chocolate is normally distributed with an unchanged standard deviation.

(6)

(Total 8 marks)

Q6 a) i) Convenience Sampling

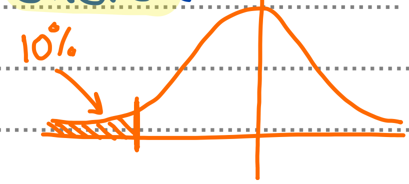
ii) This sample should not be used as it is not random. Instead it can be subject to bias.

b) State your hypotheses

$H_0 : \mu = 66.5g$

$H_1 : \mu < 66.5g$ 1 one tailed

Sketch



State the distribution

$X \sim N(66.5, 21.2^2)$ $\bar{X} = 65.4g$

Calculate the sample standard deviation $\frac{\sigma}{\sqrt{n}}$

$\frac{21.2}{\sqrt{750}}$ $\rightarrow$ new distribution $\bar{X} \sim N\left(66.5, \frac{21.2^2}{750}\right)$

Use the statistics function on your calculator

Method 1: Inverse normal

Area = 0.1

$\sigma = \dfrac{21.2}{\sqrt{750}}$

$\mu = 66.5g$

→ 65.5

Method 2: Normal CD

Lower = -10000000

Upper = 65.4

$\mu = 66.5g$

$\sigma = \dfrac{21.2}{\sqrt{750}}$

→ 0.07766

Method 3: Z-value and Standard normal

Z-value →

$$\frac{65.4 - 66.5}{21.2/\sqrt{750}}$$

$Z = -1.42$

Then use the standard normal

Area = 0.1

$\sigma = 1$

$\mu = 0$

→ -1.28

Compare the test statistic to the critical value

$65.4 < 65.5$ | $0.07766 < 0.1$ | $-1.42 < -1.28$

These are lower than the critical value therefore...

There is sufficient evidence to suggest that the advertisement has decreased the consumption of chocolate. Therefore we reject the null hypothesis

Q7.

A survey of 120 adults found that the volume, X litres per person, of carbonated drinks they consumed in a week had the following results:

$$\sum x = 165.6 \qquad \sum x^2 = 261.8$$

(a) (i) Calculate the mean of X.

(1)

(ii) Calculate the standard deviation of X.

(2)

(b) Assuming that X can be modelled by a normal distribution find

(i) $P(0.5 < X < 1.5)$

(2)

(ii) $P(X = 1)$

(1)

(c) Determine with a reason, whether a normal distribution is suitable to model this data.

(2)

(d) It is known that the volume, Y litres per person, of energy drinks consumed in a week may be modelled by a normal distribution with standard deviation 0.21

Given that $P(Y > 0.75) = 0.10$, find the value of μ, correct to three significant figures.

(4)

(Total 12 marks)

Q7 a) i) Mean $= \dfrac{\Sigma x}{n}$ $\quad \dfrac{165.6}{120} = 1.38$

ii) SD $= \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ $\quad \sqrt{\dfrac{261.8}{120} - 1.38^2} = 0.5266$

b) i) Using the normal CD

lower $= 0.5$

upper $= 1.5$

$\mu = 1.38$

$\sigma = 0.5266$

Remember to keep these accurate

$P(0.5 < X < 1.5) = 0.54278$

ii) $P(X = 1) = 0$ as the normal distribution is continuous

c) Normal distribution is suitable if n is large AND if looking at 3 standard deviations away from the mean still provides a suitable answer

$n = 120$ (greater than 30) so n is large

given $\mu = 1.38$ and $\sigma = 0.5266$ the range of values are

$$1.38 + 3(0.5266) = 2.9598$$
$$1.38 - 3(0.5266) = -0.1998$$

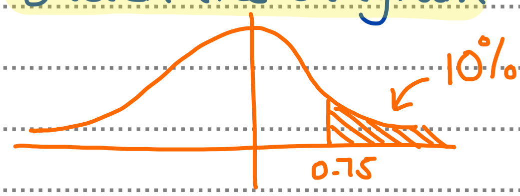
this lower interval is negative. Can it be in this context?

$-0.1998L$ of carbonated drinks is not possible, therefore the normal distribution is unsuitable

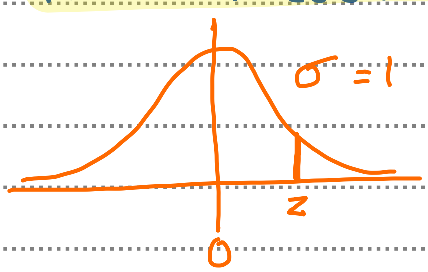
d) State the distribution $X \sim N(\mu, 0.21^2)$

$P(Y > 0.75) = 0.10$

Sketch the diagram



As we do not have the mean the standard normal needs to be found, calculating the z-value



Area $= 0.9$ (cumulatively up)
$\mu = 0$
$\sigma = 1$ $\quad z = 1.28155$

Use the z-value formula $z = \dfrac{X - \mu}{\sigma}$ $\quad 1.2816 = \dfrac{0.75 - \mu}{0.21}$

Remember 3 s.f $\rightarrow$ $\mu = 0.481$

Q8.

A teacher in a college asks her mathematics students what other subjects they are studying.

She finds that, of her 24 students:

12 study physics
8 study geography
4 study geography and physics

(a) A student is chosen at random from the class.

Determine whether the event ‘the student studies physics’ and the event ‘the student studies geography’ are independent.

(2)

(b) It is known that for the whole college:

the probability of a student studying mathematics is $\frac{1}{5}$

the probability of a student studying biology is $\frac{1}{6}$

the probability of a student studying biology given that they study mathematics is $\frac{3}{8}$

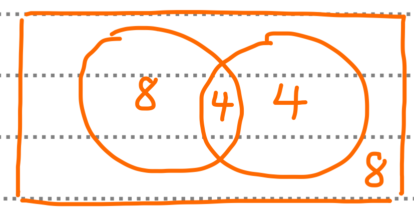
Calculate the probability that a student studies mathematics or biology or both.

(4)

(Total 6 marks)

Physics = 12 Geography = 8 Both = 4 Total = 24

Draw a picture to visualise



Test of independence → $P(A \cap B) = P(A) \times P(B)$

$P(A) \times P(B)$ $\frac{8}{24} \times \frac{12}{24} = \frac{1}{6}$

$P(A \cap B) = \frac{4}{24} = \frac{1}{6}$

$\therefore$ yes they are independent

b) $P(M) = \frac{1}{5}$ $P(B) = \frac{1}{6}$ $P(B|M) = \frac{3}{8}$

Conditional probability $= P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\therefore \dfrac{3}{8} = \dfrac{P(M \cap B)}{\frac{1}{5}} \rightarrow P(M \cap B) = \frac{3}{40}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

remember the overlap.

$P(M \cup B) = \frac{1}{5} + \frac{1}{6} - \frac{3}{40} = \frac{7}{24}$

Q9.

Abu visits his local hardware store to buy six light bulbs.

He knows that 15% of all bulbs at this store are faulty.

(a) State a distribution which can be used to model the number of faulty bulbs he buys.
(1)

(b) Find the probability that all of the bulbs he buys are faulty.
(1)

(c) Find the probability that at least two of the bulbs he buys are faulty.
(2)

(d) Find the mean of the distribution stated in part (a).
(1)

(e) State two necessary assumptions in context so that the distribution stated in part (a) is valid.
(2)

(Total 7 marks)

What distribution is it?

Q9

a) $X \sim B(6, 0.15)$

b) $P(X=6)$ $\quad X=6,\ n=6,\ p=0.15$

use binomial PD

$P(X=6) = 0.00001139$

c) $P(X \geq 2) = 1 - P(X \leq 1)$

Binomial only goes cumulatively up

$P(X \leq 1) \rightarrow X=1$

$n=6$

$p=0.15$

$P(X \leq 1) = 0.7765$

$P(X \geq 2) = 1 - 0.7765$

$= 0.2235$

d) Mean in the binomial distribution = np

$0.15 \times 6 = 0.9$

e) ① Events are independent

② The probability of a faulty bulb stays the same

Q10.

Of the patients in an accident unit, 30% have a sports injury.

Past records for the accident unit suggest that:

a patient with a sports injury has a probability of 0.2 of being admitted to hospital;

a patient who does **not** have a sports injury has a probability of 0.4 of being admitted to hospital.

A patient is chosen at random from those in the accident unit.

(a) (i) Write down the probability that the chosen patient has a sports injury.

(1)

(ii) Find the probability that the chosen patient has a sports injury and is admitted to hospital.

(1)

(iii) Show that the probability that the chosen patient is admitted to hospital is 0.34.

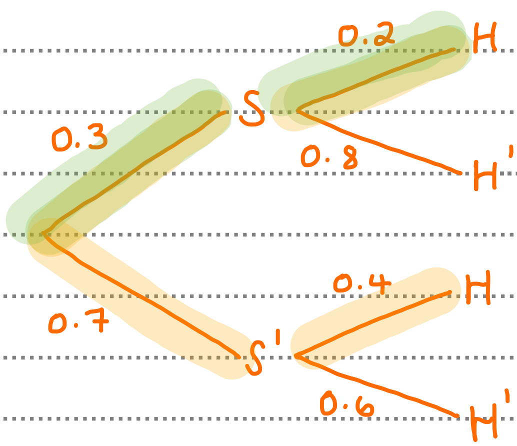
(2)

(b) Given that the chosen patient is admitted to hospital, find the conditional probability that the patient has a sports injury.

(3)

(Total 7 marks)

Set the scene and draw a diagram



$S \rightarrow$ sports injury

$H \rightarrow$ admitted

a)

i) $P(S) = 0.3$

ii) $P(S \cap H) = 0.3 \times 0.2 = 0.06$

iii) Admitted to hospital

$P(S \cap H) = 0.06$

OR

$P(S' \cap H) = 0.7 \times 0.4$

$= 0.28$

$= 0.34$

b) $P(S|H) = \dfrac{P(S \cap H)}{P(H)}$

Use your facts from part a)

$= \dfrac{0.06}{0.34} = 0.1765$